CS 6170 Computational Topology: Topological Data Analysis | **University of Utah**
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24.1 Elevation Function

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Elevation Function E: X $\rightarrow$ R.
For x $\in$ X, E(x) is the persistence of x when it becomes critical.

Definition 24.1. *N(x) is the set of directions s.t. x is a critical point. (normal directions)*

Example:
Figure 24.1. shows the case for 1-manifold. For this simple case,

- E(a) = E(f) = $\|t_f - t_a\|$
- E(b) = E(c) = $\|t_c - t_b\|$
- E(d) = E(e) = $\|t_e - t_d\|$

The extended persistent pairs are (a,f), (b,c), (d,e).

Figure 24.2. shows the two-legged case. In this case, both c and d have the same height and both can be paired with b or a, we need to slightly perturb h to h', then the corresponding c' and d' will have different heights. c' becomes the global maximum and can be paired with a'. d' can be paired with b'. Then the pairing is no longer ambiguous. The new persistent pair are (a',c') and (b', d'). This is a degenerate case.

24.1.2 Piecewise Linear Approximation

In real cases, we cannot generate smooth continuous functions, thus a piecewise linear approximation of manifold is used instead. In these cases, normal direction is not a single direction anymore, and we need to look at the adjacent points for the normal directions, which can be shown by Figure 24.3(a). For the continuous manifold, the normal direction of a critical point x' is represented by N(x'). However, for the piecewise linear approximation case, we need to rely on the neighbor points of X, such as a and b. Normal directions N(x) are shown in Figure 24.3(a). Figure 24.3.(b) shows the normal direction for piecewise linear approximation case. For two critical points x and y, the normal directions are represented by $u = \dfrac{x-y}{\|x-y\|}$ and $u \in N(x) \wedge N(y)$. Thus for an approximation with n vertices, the number of normal directions is C_n^2.

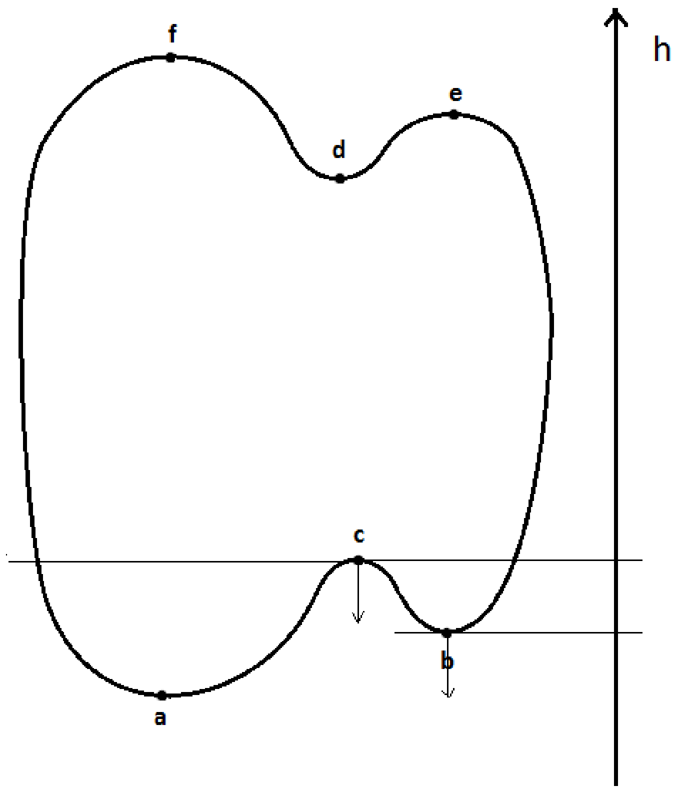


Figure 24.1: One Legged Case

24.2 Reeb Space

Reeb Space is the extension of Reeb Graph to multi-dimension.
f: $X \to R^d$.

Definition 24.2. *For x, y ∈ X, x and y are equivalent (x ∼ y) if:*

- *f(x) = f(y).*
- *x, y ∈ same path connected component of* $f^{-1}(a)$.

Definition 24.3. *Reeb Space is the Quotient Space obtained by identifying equivalent points. R(X,f) = X/∼*

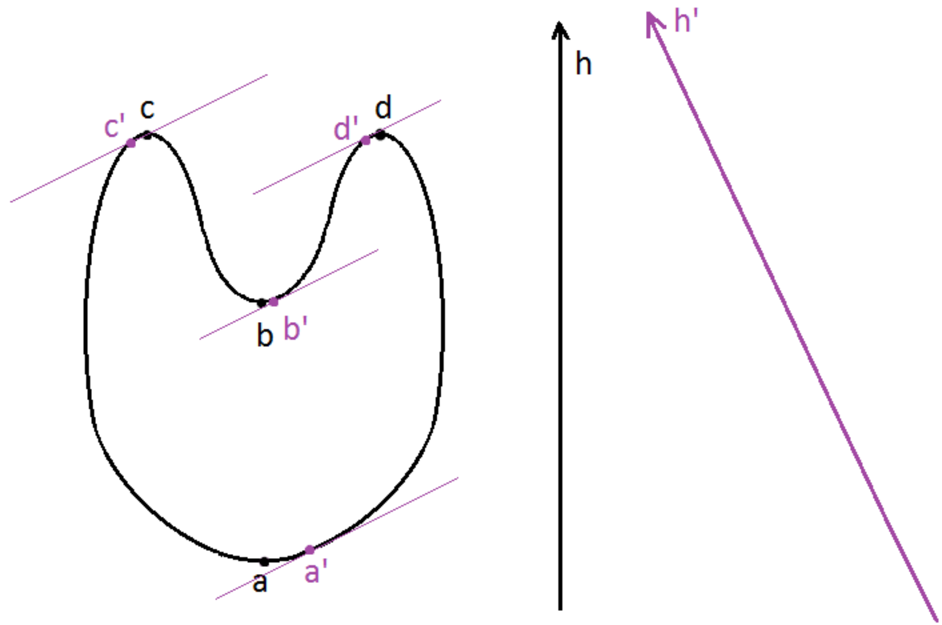
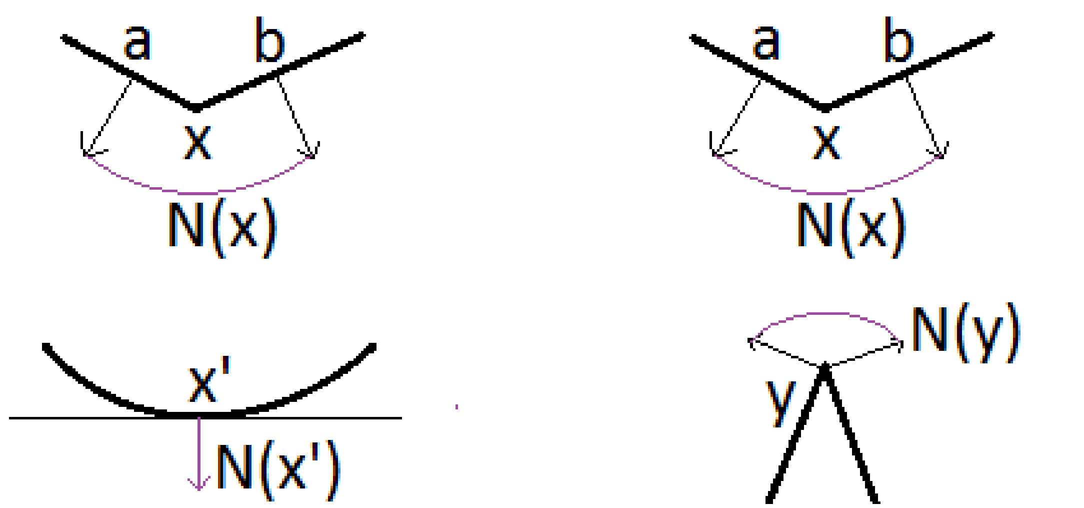


Figure 24.2: Two Legged Case



(a) Comparison Between Continuous and Discrete Normal

(b) Normal Directions for PL Approximation

Figure 24.3: Piecewise Linear Approximation