CS 6170 Computational Topology: Topological Data Analysis **University of Utah**
Spring 2017 **School of Computing**

Lecture 21: March 28, 2017

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Final Project Requirement: report in pdf format

presentation: May 1 - May 3

21.1 Overview

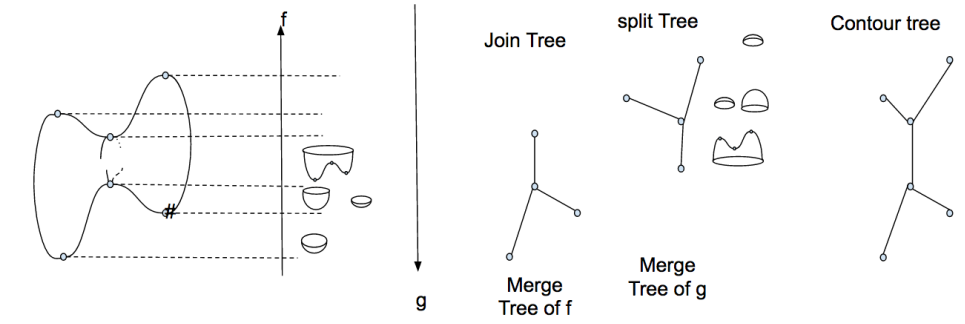
This class go through two important concept:mapper, contour tree.

Contour Trees:is a graph that tracks component of the level set as they split and appear or join and disappear[Carr2003].

Mapper. $f : X \to Z$ (Z could be R, R^2...) and $u = \{U_\alpha\}_{\alpha \in A}$ be a finite open cover of z. mapper is the pullback cover $M(u, f) = N(f^*(u))$ where $f^*(u)$ is defined as connected component of $f^-1(u_\alpha)$

21.2 Contour Tree

let's say we have a topological space X and $f : X \to R$ and $g = -f$. Base on the function of f we can construct a tree from bottom to top called Join Tree. Function g is opposite of f which construct the split tree. merge tree of f combine with merge tree of g gives us a reeb graph / contour tree.



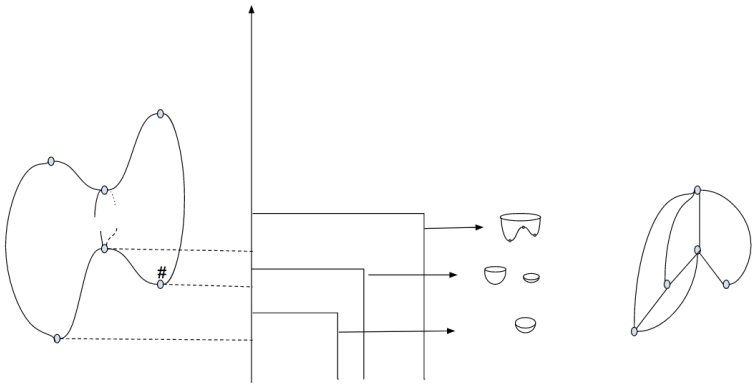
In the original paper[Carr2003], they give a more detail description about the contour tree (I just copy them here)

- each leaf vertex represents the creation or deletion of a component at a local extremum of the parameter
- each interior vertex represents the joining and /or splitting of two or more components at a critical point

- each edge represents a component in the level sets for all values of the parameter between the value of the data points at each end of the edge.

21.3 Sub-Level Set

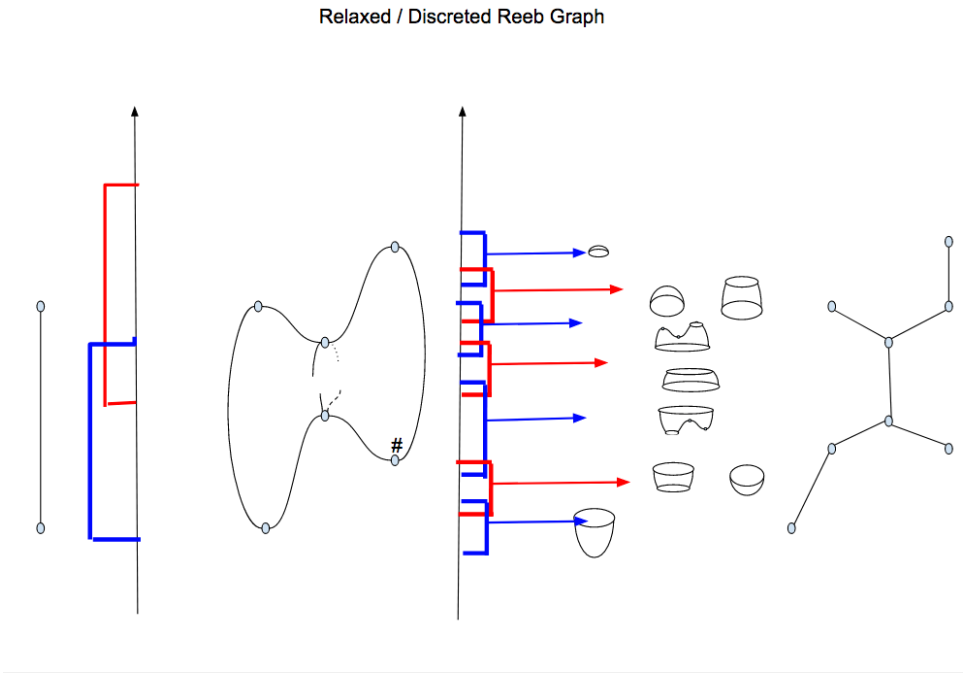
For each set, there should a connection between itself and its sub level set, which lead to the new graph base on join tree.



21.3 Intervel

For the same object, if we use different resolution(eg. interval) which will lead to very different looking of the graph. Check the following graph as one example

- If intervals are too large, we'll miss all topological structure(the graph on the left side)
- If intervals are too small, although the intervals of U_α are interesting the intervals of pull-back cover may not overlap (this is because we usually work with finite sample data)
- There a a few parameter that you can think about it:
  - size of cover element
  - overlap: eg:20%(the region cover by blue and red)
  - function



21.4 Using Mapper

There are a few parameters may think about when we are using mapper

1 Data (point could, triangulation)

2 Function (height, CLV, labels, curvature, eccentricity etc.)

3 Metric (How to define distance between two points? eg. euclidean, hamming distance)

4 Clustering technique: Hierarchical(single-link, max-link), k-means

5 Interval

6 Overlap

References

[Carr2003] CARR, HAMISH,JACK SNOEYINK and ULRIKE AXEM, "Computing contour trees in all dimensions." *Computational Geometry 75-94.*, 2003