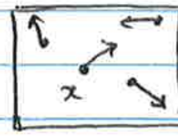
Apr 13

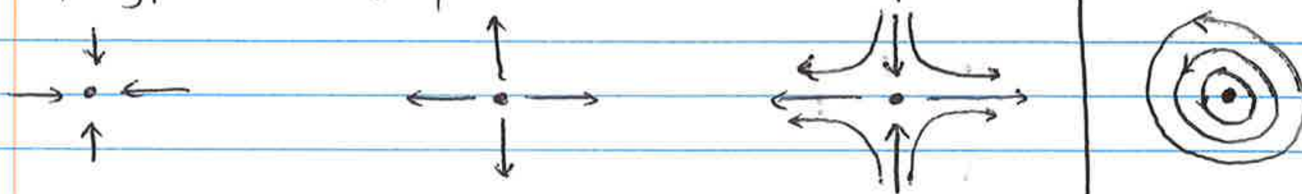
TDA + Vector Field

Example $f: \mathbb{R}^2 \to \mathbb{R}^2$



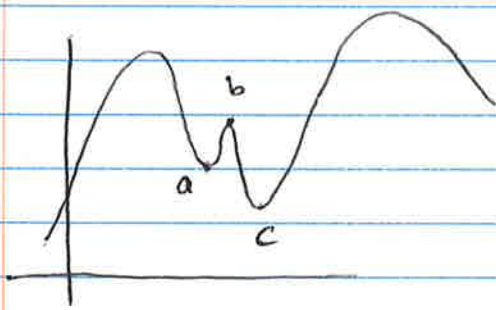
There is a vector attached to each location in plane.

4 types of simple / 1^{st} order critical points $\boxed{\nabla f = 0}$



Sink Source Saddle Center

#



(a,b) is a persistence pair. The persistence of the pair is the amount of perturbation that will eliminate the pair.

Well Group Theory: $f: X \to Y$, $A \subseteq Y$

X, Y are manifolds, A could be sub-mnfld.

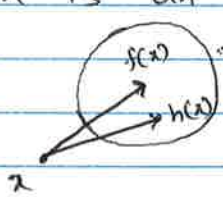
Stability / Robustness of $H(f^{-1}(A))$ w.r.t. perturbation of f.

eg. $f: \mathbb{R}^2 \to \mathbb{R}^2$, $A = \{0\} \subseteq \mathbb{R}^2$. We want to study stability of $H(f^{-1}(A)) \Rightarrow f^{-1}(A)$ is a critical point of f

$f^{-1}(0)$: Critical points of f.

→ r-perturbation: $f, h : \mathbb{R}^2 \to \mathbb{R}^2$, $d(f,h) = \sup_{x \in \mathbb{R}^2} \|f(x) - h(x)\|_2$

h is an r-perturbation of f if $d(f,h) \le r$



→ ball of radius r.

→ $f_0 = \|f\|_2$, $F_r = f_0^{-1}[0, r]$

Every point inside the shaded region has magnitude $< r$ ←

Every point on the boundary has magnitude r

→ F_r : Sub-level sets of magnitude field.

→ $i : h^{-1}(0) \rightarrow F_r$ where h is an r-perturbation of f

↳ inclusion map : if you perturb f by r, critical points are changed by magnitude $\leq r$.

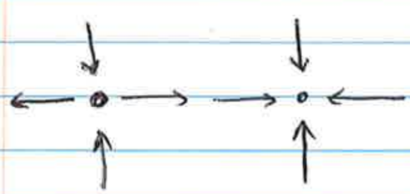
→ $j_h : H_1(h^{-1}(0)) \longrightarrow H_1(F_r)$

→ Well group $U(r) = \bigcap_{\substack{r\text{-pert.} \\ h \text{ of } f}} \text{image}(j_h)$

$U(r)$ is a sub-group of the homology group of F_r.

Intersection of all possible r-perturbations of f : infinite possibilities.

efficient computation cases : $A = \{a\} \subseteq \mathbb{R}$ or $A = [a, b] \subseteq \mathbb{R}$



Saddle Sink

→ perturbing the flow along the connection cancels out ~~the~~ both the critical points

after perturbation.

C.P.	degree
Sink	+1
Source	+1
Saddle	−1

Sink - saddle
Source - sadde } combined degree of the region is 0

→ Critical points can be eliminated.