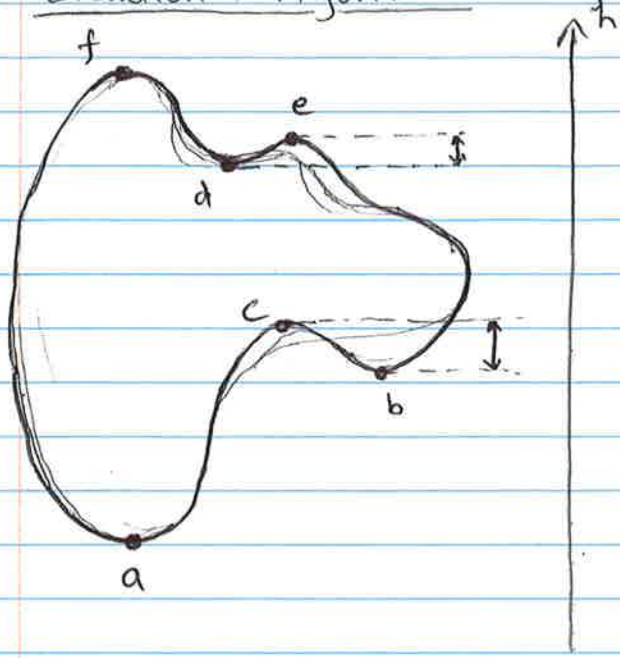
Apr 6

Elevation : Algorithm



smooth 1-manifold. $\mathbb{X}$

$E : \mathbb{X} \to \mathbb{R}$

$E(x) =$ persistence (x)

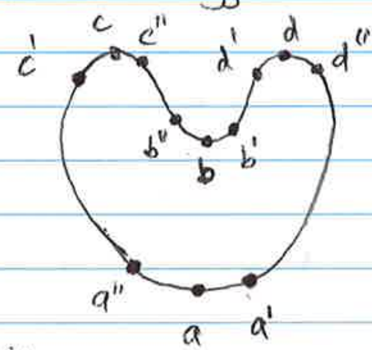
(a, f)

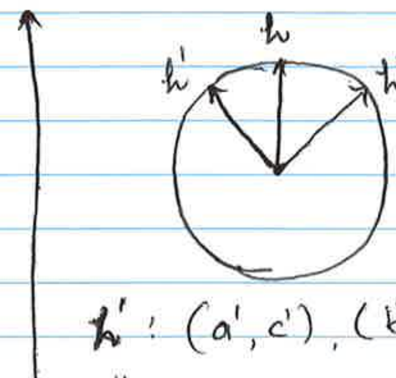
(b, c)

(d, e)

1 - legged case

2 - legged Case





h' : (a', c'), (b', d')

h'' : (a'', d''), (b'', c'')

For 2-legged case with direction h, both c and d have same height ~~to~~ so the pairing is ambiguous.

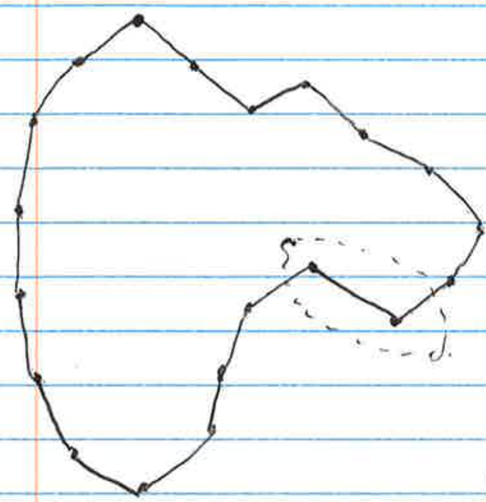
But small perturbation in direction makes pairing un-ambiguous.

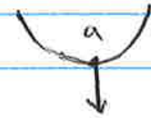
Def: $N(x)$: Set of directions s.t. x is a critical point.

$N(x, y)$: Intersection : $N(x) \cap N(y)$

$N(x, y, \ldots)$: $N(x) \cap N(y) \cap \ldots$

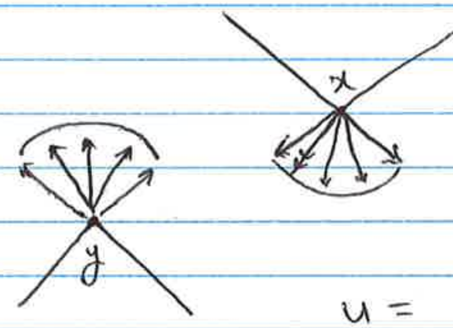
Piecewise linear approximation





in continuous case, theres is only one normal direction for a.

In case of piecewise linear approximation:

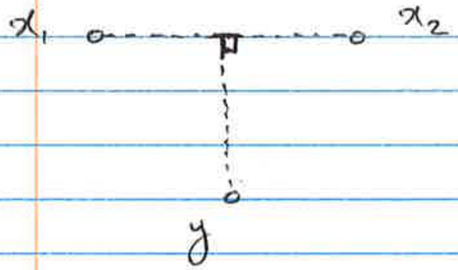


all these are valid normals by our definition.

$$u = \frac{x-y}{\|x-y\|}, \quad u \in N(x,y)$$

$$N(x) \cap N(y)$$

In 2-legged case:



① $z \in x_1, x_2$

② $u = \frac{y-z}{\|y-z\|}$

$$u \in N(x_1, x_2, y)$$

Naive Algorithm: process all possible cases.

$\boxed{O(N^4)}$

in 1-legged case: $\binom{N}{2}$ possible pairs

in 2-legged case: $\binom{n}{3}$ possible triples of points.

for 3-legged and 4-legged cases $\binom{N}{4}$ possible combinations of points.

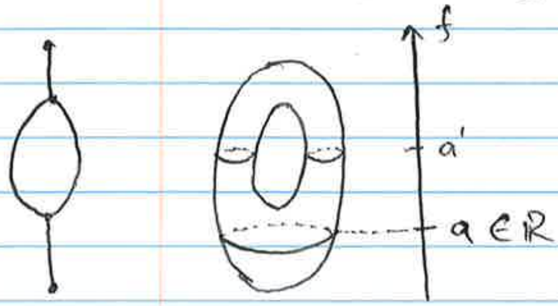
N: numbers of vertices in PL approximation.

⇒ In practice, we can prune number of cases to be considered. in 1-legged, 1-manifold case: Compute normal arcs of all points in advanced and prune ~~~~ pairs where normal directions have no intersections. → same can be done for 2-legged case.

Reeb Space

$f : \mathbb{X} \to \mathbb{R}^d$, a generic continuous function.

→ $x, y \in \mathbb{X}$ are "equivalent" if ① $f(x) = f(y)$ and ② x, y belong to same path connected component of $f^{-1}(a)$



Reeb space is the quotient space obtained by ~~identified by~~ identifying equivalent points.

$$R(\mathbb{X}, f) = \mathbb{X}/\sim$$

all points in this level set are equivalent.

$d = 1$ is Reeb graph
special case of Reeb space.