Review: Mapper. $f : X \to Z$ (Z could be $\mathbb{R}$, $\mathbb{R}^2$ etc.)

$\mathcal{U} = \{U_\alpha\}_{\alpha \in A}$ be a finite open cover of Z

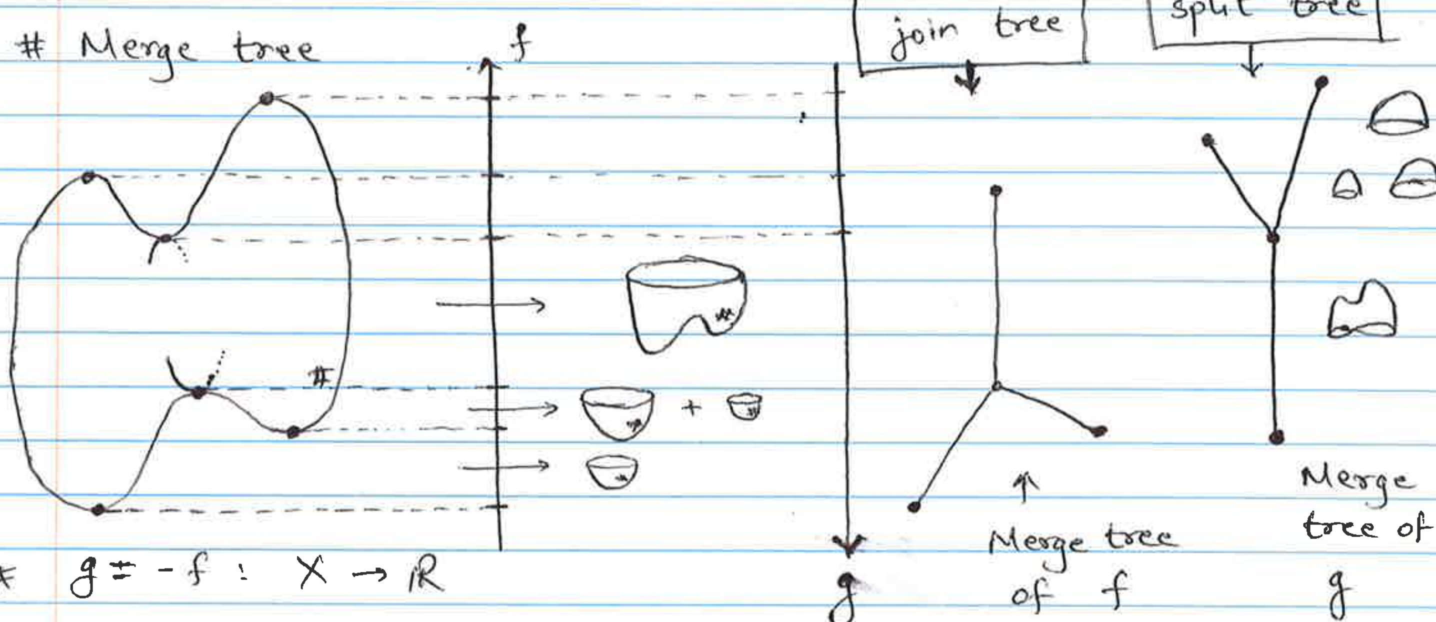
mapper is the pull-back cover [Nerve of f^* : $N(f^*)$]

$$M(\mathcal{U}, f) := N(f^*(\mathcal{U}))$$

where

$f^*(\mathcal{U})$ is defined as connected component of $f^{-1}(U_\alpha)$

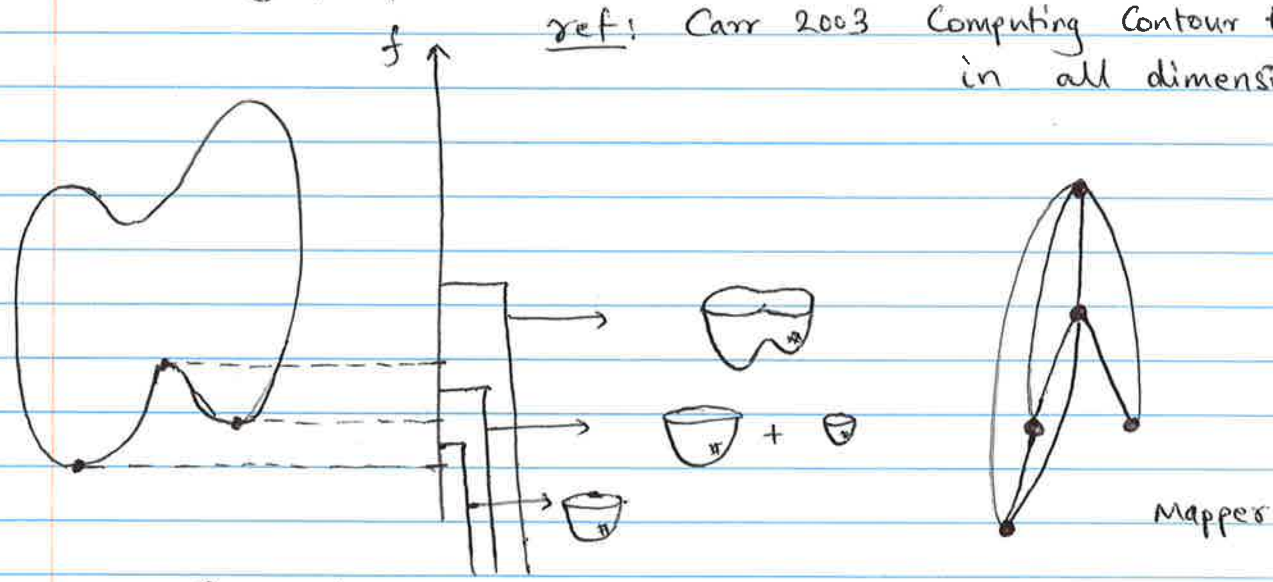
Merge tree



$g = -f : X \to \mathbb{R}$

→ Merge tree of f + Merge tree of g gives us the Reeb graph / contour tree

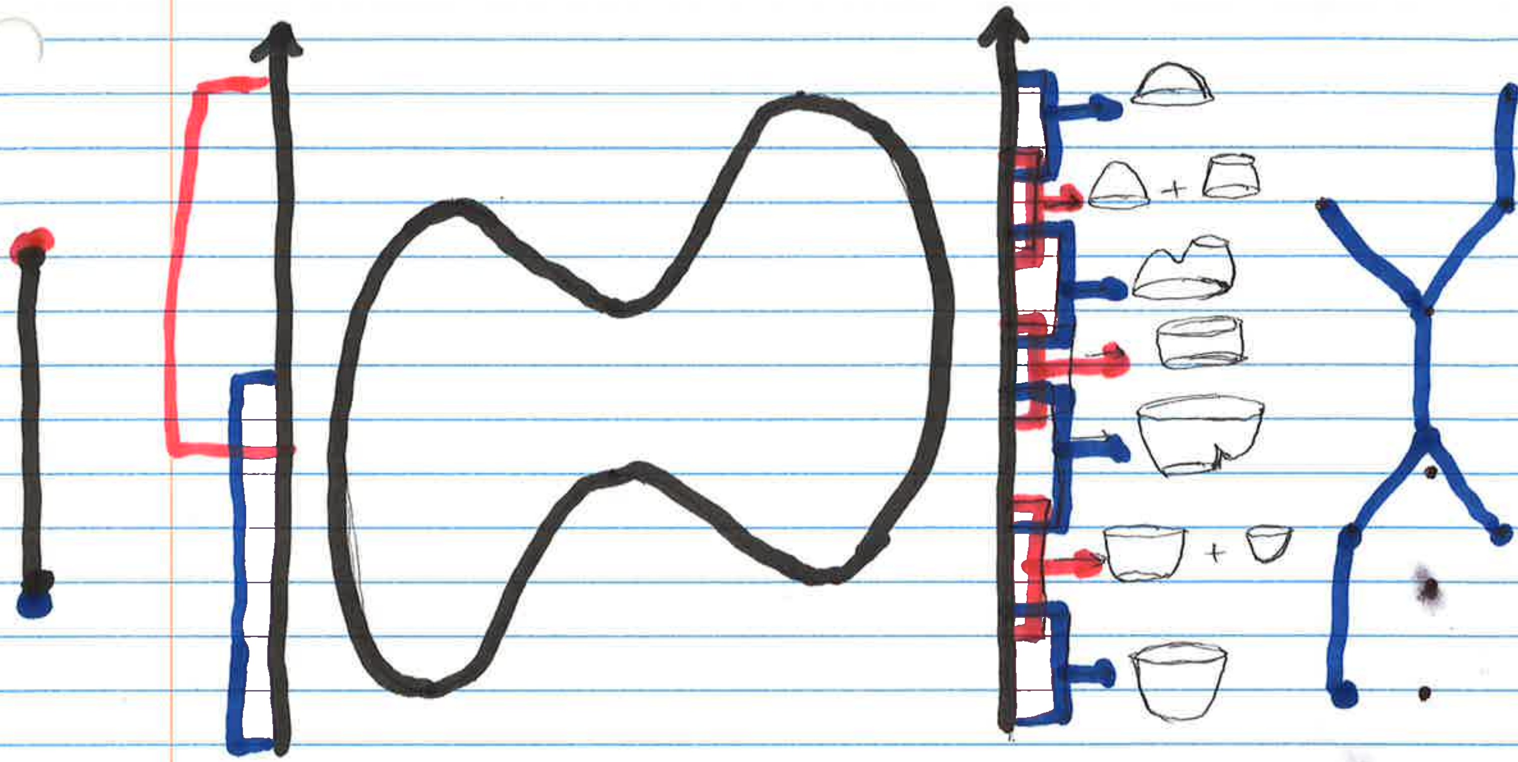
ref: Carr 2003 Computing Contour tree in all dimensions



$U_a = (-\infty, a)$

$f^{-1}(-\infty, a) = f^{-1}(U_\alpha)$ → sub-level sets

Relaxed / discretized Reeb graph



⇒ Same object, different resolutions lead to very different looking graphs

We need the right resolution for open cover U_α

if intervals are too large, we'll miss all topological structure (the graph on left)

if intervals are too small: although the intervals of U_α are intersecting the intervals of pull-back cover may not overlap (this is because we usually work with finite, sampled data)

Using Mapper

① Data? (PCD, triangulation)

② function? (height, CLV, Labels, curvature eccentricity etc.)

③ Metric? (How to define distance betn two points? eg. Euclidean, Hamming distance)

④ Clustering technique Hierarchical, K-means etc.

⑤ Interval size

⑥ Overlap between intervals.