A AN EXAMPLE OF MERGE TREE

We now give an example of an augmented merge tree that is constructed from the ERA5 Wind dataset. First, we define a scalar field $f_0 : \mathbb{X} \to \mathbb{R}$ by assigning the vector magnitude to each point $x \in \mathbb{X}$, that is, $f_0(x) = ||f(x)||_2$; see Fig. 12(A) and (B), which visualize the vector field f and scalar field f_0. Second, we track the merging behavior of components that contain critical points of f. For example, the component containing x_1 and x_2 merges with the component containing x_3 at $r = 0.84$ and forms C_3 in $\mathbb{X}_{0.84}$, which is represented by the purple and green region in Fig. 12 (D). Third, we augment the merge tree with the degrees of critical points (on leaves) and the degrees of components (on internal nodes). For example, component C_1 in Fig. 12 (D) contains critical points x_1 and x_2, whose degrees are $+1$ and -1, respectively. The degree of C_1 is $\deg(x_1) + \deg(x_2) = 0$, i.e., $\deg(C_1) = 0$. The augmented merge tree of Fig. 12 (A) is shown in Fig. 12 (E).

B AN EXAMPLE OF ROBUSTNESS CALCULATION

Using the example in Fig. 12 (A)-(E), we now show how to calculate robustness with an augmented merge tree. As pointed out in Sec. 3.1, the robustness of a critical point can be calculated as the function value of its lowest zero-degree ancestor in the augmented merge tree. The robustness of x_1 and x_2 is 0.65, whereas the robustness of x_3 and x_4 is 14.7. Intuitively, for the example in Fig. 12 (A)-(E), it is easier for x_1 and x_2 to be canceled with each other than x_3 and x_4, since they have much lower robustness values. In Fig. 12 (F), we give the vector field from the same dataset but one time step (6 hours) behind the vector field of Fig. 12 (A). We see x_1 and x_2 disappear, whereas x_3 and x_4 remain.

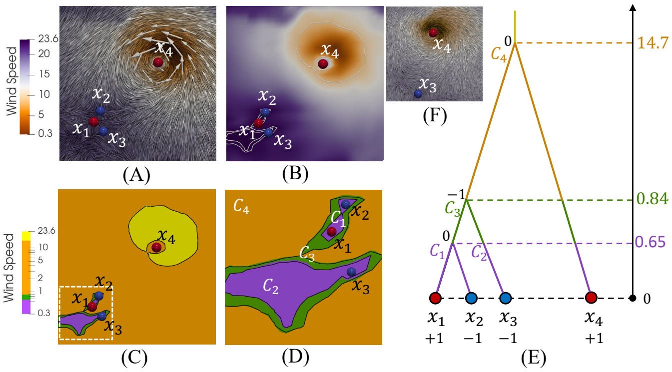

Fig. 12: Computing robustness with an augmented merge tree. (A) A 2D vector field f and (B) its corresponding scalar field f_0. (C) Multiple sublevel sets of f_0. (D) A zoomed-in view of the white box in (C). (E) The augmented merge tree. (F) The vector field one time step behind (A). Sources/sinks/centers are in red, and saddles are in blue.

C DETAILS ON DATASET AND METHODS

We demonstrate the performance of TROPHY using 30-year (1981–2010) near-surface wind vector field from the ECMWF Reanalysis v5 (ERA5). It is produced by the Copernicus Climate Change Service (C3S) [1]. ERA5 provides hourly estimates of the global climate information with a spatial grid resolution of 30 km. Since tropical cyclones/storms usually occur during June and October, we limit our dataset with a time window from June 1 to October 31 every year at standard synoptic reporting times (0000, 0600, 1200, and 1800 UTC). A rectangle region on the Atlantic Ocean (5 N° to 49.5 N° and 98 W° to 18 W°) is selected. We utilize 10-meter zonal and meridional wind speed as the 2D vector field, since in the near-surface the hurricane core represents a region of strong convergence and associated vertical motion. We annotate this 30-year dataset as the ERA5 Wind dataset. We also mark the one-year subset data from the ERA5 Wind dataset as ERA5 Year; for example, Fig. 3 uses the ERA5 2004 dataset.

We use the International Best Track Archive for Climate Stewardship (IBTrACS [26] version 4) observations as the reference. IBTrACS is compiled from quality-controlled records from various forecasting centers. In this paper, we select TCs reported by World Meteorological Organization (WMO) official forecast centers. Again, only tropical cyclones/storms within the region of the ERA5 Wind dataset are visualized. For comparison purposes, we include the TC tracking results of the TempestExtremes software package [44] applied to the ERA5 Wind dataset with the parameter setting following [55].