Image representation, sampling and quantization

António R. C. Paiva

ECE 6962 – Fall 2010

Lecture outline

Image representation

Digitalization of images

Changes in resolution

Matlab tutorial

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Image as a function I

- *An image is a function of the space.*

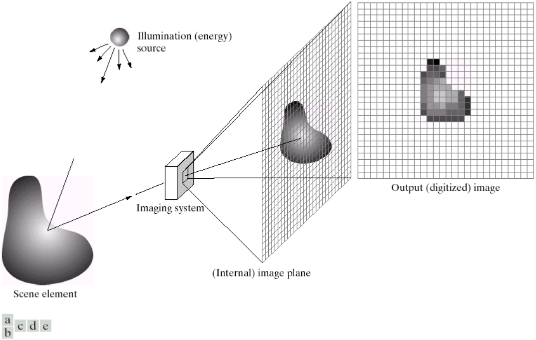
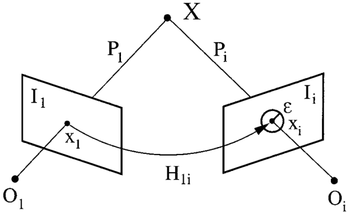


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

- Typically, a 2-D projection of the 3-D space is used, but the image can exist in the 3-D space directly.

Image as a function II

- The fact that a 2-D image is a projection of a 3-D function is very important in some applications.



(From Schmidt, Mohr and Bauckhage, IJCV, 2000.)

- This in important in image stitching, for example, where the structure of the projection can be used to constrain the image transformation from different view points.

Image as a *single-valued* function

- The function can be single-valued

$$f : \mathbb{R}^m \longrightarrow \mathbb{R}, \quad m = 2, 3,$$

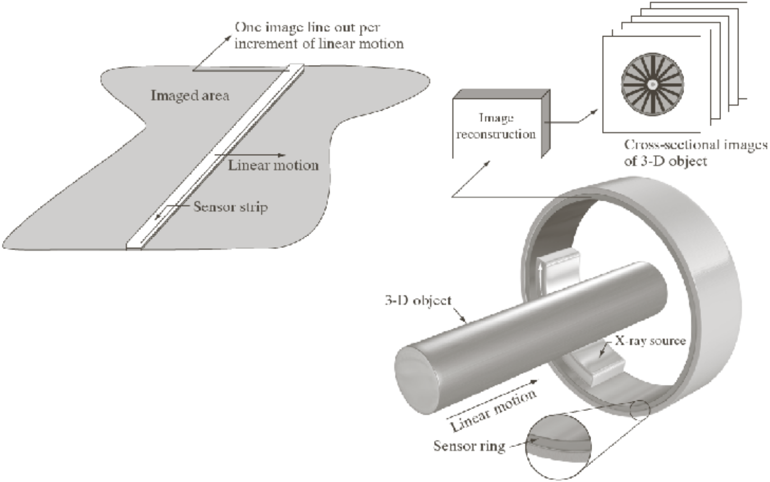
quantifying, for example, intensity.

Image as a *multi-valued* function

- …or, be multi-valued, $f : \mathbb{R}^m \longrightarrow \mathbb{R}^3, m = 2, 3.$
  The multiple values may correspond to different color intensities, for example.

Red Green Blue

2-D vs. 3-D images



Images are *analog*

- Notice that we defined images as functions in a continuous domain.
- Images are representations of an analog world.
- Hence, as with all digital signal processing, we need to digitize our images.

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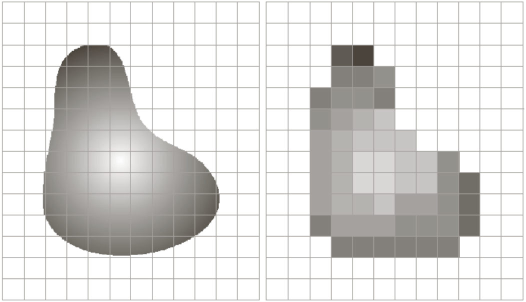
Digitalization

- Digitalization of an analog signal involves two operations:
  - Sampling, and
  - Quantization.
- Both operations correspond to a discretization of a quantity, but in different domains.

Sampling I

- Sampling corresponds to a discretization of the space. That is, of the domain of the function, into $f : [1, \ldots, N] \times [1, \ldots, M] \longrightarrow \mathbb{R}^m$.



a b

FIGURE 2.17 (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

Sampling II

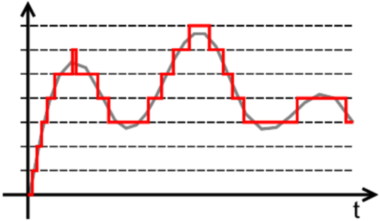
- Thus, the image can be seen as matrix,

$$f = \begin{bmatrix} f(1,1) & f(1,2) & \cdots & f(1,M) \\ f(2,1) & f(2,2) & \cdots & f(2,M) \\ \vdots & \vdots & \ddots & \vdots \\ f(N,1) & f(N,2) & \cdots & f(N,M) \end{bmatrix}.$$

- The smallest element resulting from the discretization of the space is called a pixel (picture element).
- For 3-D images, this element is called a voxel (volumetric pixel).

Quantization I

- Quantization corresponds to a discretization of the intensity values. That is, of the co-domain of the function.

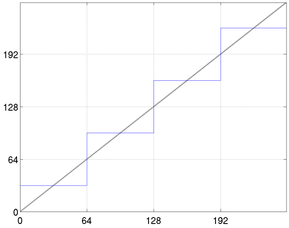


- After sampling and quantization, we get $f : [1, \ldots, N] \times [1, \ldots, M] \longrightarrow [0, \ldots, L]$.

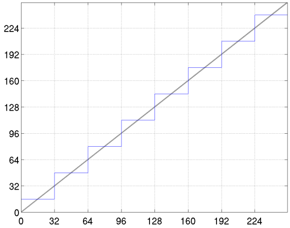
Quantization II

- Quantization corresponds to a transformation $Q(f)$

4 levels

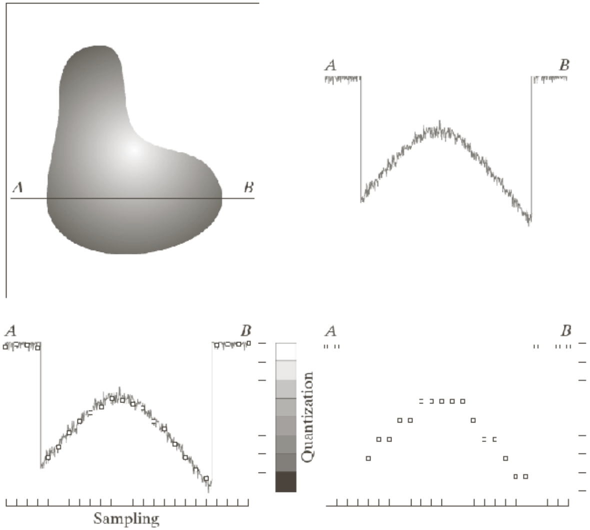


8 levels



- Typically, 256 levels (8 bits/pixel) suffices to represent the intensity. For color images, 256 levels are usually used for each color intensity.

Digitalization: summary



a b
c d

FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

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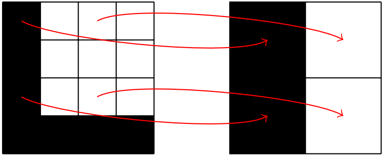
Which resolution?

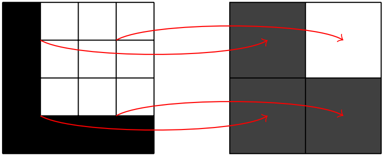
- Digital image implies the discretization of both spatial and intensity values. The notion of resolution is valid in either domain.
- Most often it refers to the resolution in sampling.
  - Extend the principles of multi-rate processing from standard digital signal processing.
- It also can refer to the number of quantization levels.

Reduction in sampling resolution I

- Two possibilities:
  - Downsampling



  - Decimation



Reduction in sampling resolution II

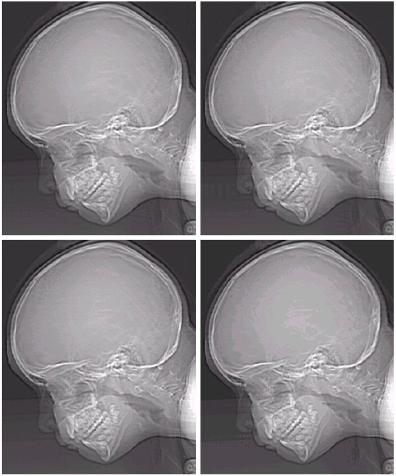
a b
c d

FIGURE 2.20 Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.

Increase in sampling resolution

- The main idea is to use interpolation.
- Common methods are:
  - Nearest neighbor
  - Bilinear interpolation
  - Bicubic interpolation

Decrease in quantization levels I



a b
c d

FIGURE 2.21
(a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.

Decrease in quantization levels II

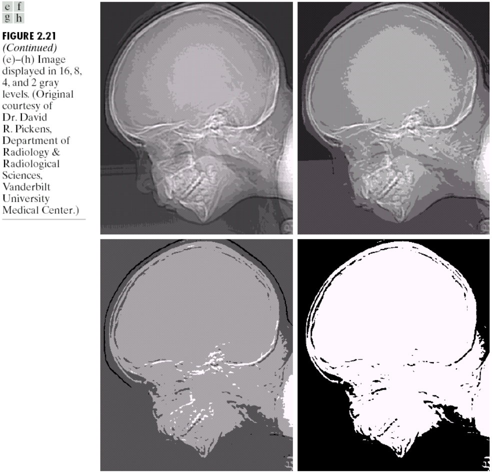
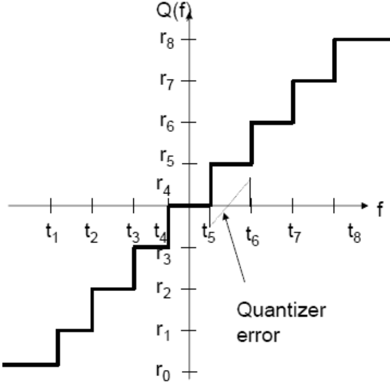


FIGURE 2.21 *(Continued)* (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)

Non-uniform quantization I

- The previous approach considers that all values are equally important and uniformly distributed.



Non-uniform quantization II

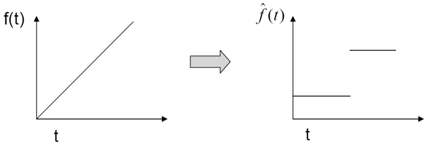
- What to do if some values are more important than others?
- In general, we can look for quantization levels that “more accurately” represent the data.
- To minimize the mean square error (MSE) we can use the Max-Lloyd algorithm to find the quantization levels with minimum MSE.

Non-uniform quantization III

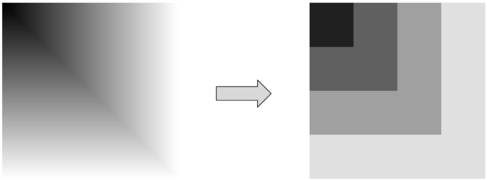
- Max-Lloyd algorithm:
  1. Choose initial quantization levels;
  2. Assign points to a quantization level and reconstruct image;
  3. Compute the new quantization levels as the mean of the value of all points assigned to each quantization level.
  4. Go back to 2 until reduction of MSE is minimal.

The “false contour” effect I

- By quantizing the images we introduce discontinuities in the image intensities which look like contours.
  - in 1-D,



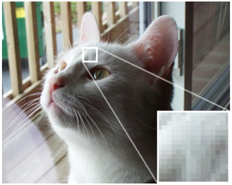
  - in 2-D,



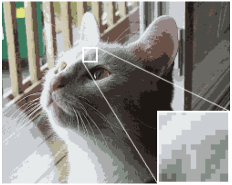
The “false contour” effect II

- To mitigate the “false contour” effect we can use dither.
  - Basically, we add noise before quantization to create a more natural distribution of the new intensity values.

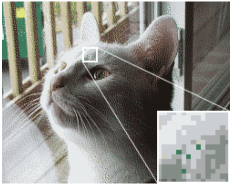
Original



Undithered



Dithered



(Images from Wikipedia.)

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Reading images

- Use imread to read an image into Matlab:

```
» img = imread('peppers.jpg','jpg');
» whos
  Name        Size                  Bytes  Class
  img         512x512x3            786432  uint8
```

  - Format is: `A = IMREAD(FILENAME,FMT)`.
    Check the help, `help imread`, for details.
  - Note that data class is `uint8`.
    Convert to double with `img = double(img);`.
    This is necessary for arithmetic operations.

Displaying images I

- With Image Processing toolbox:
  use imshow to display the image.

```
» imshow(img);
» imshow(img(:,:,1));
    % Shows only the red component of the image
```

  - The image must be in `uint8` or, if double, normalized from 0 to 1.

Displaying images II

- Without the Image Processing toolbox:
  use image to display the image.

  » **image**(img);

  - The image must have 3 planes.
    So, for grayscale images do,

    » **image**(repmat(gray_img, [1 1 3]));

Saving images

- Use imwrite to save an image from Matlab:

```
» imwrite(img,'peppers2.jpg','jpg');
» imwrite(img(:,:,1),'peppersR.jpg','jpg');
  % Saves only the red component of the image
```

  - Format is: `IMWRITE(A,FILENAME,FMT)`.
    Check the help, `help imwrite`, for details.
  - The image should be in `uint8` or,
    if double, normalized from 0 to 1.

Reading

- Sections 2.4 and 2.5 of the textbook.